Turn in the following problems:

1. Prove that
$$\lim_{x\to\infty} \frac{\ln(x)}{x^p} = 0$$
for any number $p > 0$. This shows that the logarithmic function approaches ∞ more slowly than any power of x.

2. If an object with mass m is dropped from rest, one model for its speed v after t seconds, taking air resistance into account is
$$v = \frac{mg}{c}(1 - e^{-ct/m})$$
where g is the acceleration due to gravity and c is a positive constant. (In Calculus 2 we will be able to deduce this equation from the assumption that the air resistance is proportional to the speed of the object; c is the proportionality constant.)
   (a) Calculate $\lim_{t\to\infty} v$. What is the meaning of this limit?
   (b) For fixed t, use l'Hôpital's Rule to calculate $\lim_{c\to 0^+} v$. What can you conclude about the velocity of a falling object in a vacuum?

3. Recall that the Extreme Value Theorem guarantees that continuous functions have global maxima and global minima over every closed, bounded interval.

   Consider the following mathematical statements. Fill in the blank with "all", "no", or "some" to make the following statements true. Note that "some" means one or more instances, but not all.

   - If your answer is "all", then give a brief explanation as to why.
   - If your answer is "no", then give an example and a brief explanation as to why.
   - If your answer is "some", then give two specific examples that illustrate why your answer it not "all" or "no". Be sure to explain your two examples.

   (a) For _______ real numbers b, if $f(x) = x^2$, then f has a global maximum on the interval $(0, b)$.
   (b) For _______ functions f, if f is differentiable and has a global maximum on the interval $0 \le x \le 10$, then $f'(x) = 0$ for some x in the interval $(0, 10)$.
   (c) For _______ functions f, if f is continuous and differentiable on $0 \le x \le 5$ and f has exactly one critical point at $x = 3$, then f has either a global maximum or minimum at $x = 3$.

4. A rectangular storage container with an open top is to have a volume of 10 m^3. Then length of its base is twice the width. Material for the base costs \$10 per square meter. Material for the sides cost \$6 per square meter. Find the cost of materials for the cheapest such container.

5. A cylindrical can without a top is made to contain a volume of 2000 cubic centimeters of liquid. Find the dimensions that will minimize the cost of the metal to make the can.

6. The manager of a 100-unit apartment complex knows from experience that all units will be occupied if the rent is \$800 per month. A market survey suggests that, on average, one additional unit will remain vacant for each \$10 increase in rent. What rent should the manager charge to maximize revenue?

These problems will not be collected, but you might need the solutions during the semester:

7. A boat leaves a dock at 2:00 PM and travels due south at a speed of 20 km/h. Another boat has been heading due east at 15 km/h and reaches the same dock at 3:00 PM. At what time were the two boats closest together?

8. Find the limit. Use l'Hôpital's Rule if appropriate. If there is a more elementary method, consider using it. If l'Hôpital's Rule does not apply, explain why.

$$\lim_{x\to\infty}\left(1+\frac{a}{x}\right)^{bx}$$

9. Use l'Hôpital's Rule to help find the asymptotes of f. Then use them, together with the information from f' and f'', to sketch the graph. Check your work with a graphing device or program.

$$f(x) = xe^{-x^2}$$

10. If an object with mass m is dropped from rest, one model for its speed v after t seconds, taking air resistance into account is

$$v = \frac{mg}{c}(1 - e^{-ct/m})$$

where g is the acceleration due to gravity and c is a positive constant. (In Calculus 2 we will be able to deduce this equation from the assumption that the air resistance is proportional to the speed of the object; c is the proportionality constant.)

   (a) Calculate $\lim_{t\to\infty} v$. What is the meaning of this limit?

   (b) For fixed t, use l'Hôpital's Rule to calculate $\lim_{c\to 0^+} v$. What can you conclude about the velocity of a falling object in a vacuum?

11. If f' is continuous, $f(2) = 0$, and $f'(2) = 7$, evaluate

$$\lim_{x\to 0}\frac{f(2+3x)+f(2+5x)}{x}$$

Optional Challenge Problem

This problem will help improve your algebra skills.

Graph the function $f(x) = (x-2)^{1/3}x^{2/3}$. Include calculation of the first and second derivatives and full analysis of the increasing/decreasing behavior, local extrema, concavity and inflection points, and the end behavior.